

**KAKATIYA GOVERNMENT COLLEGE**  
**HANUMAKONDA**

---

Name : Dr. B. Prabhakar  
Designation : Asst. Prof of Mathematics  
Year of Award of PhD. : 2017  
Name of the University : Osmania University, Hyderabad  
Year of entering into Govt. Service : 24-12-2011.

S. No.	Details of copies of Certificates	Remarks
1	Copy of Ph.D Certificate	Enclosed
2	Press note	Enclosed
3	Research work dates of seminars and Pre-Ph.D Date of joining in this college	Enclosed 04-7-2017
4	Details of Ph.D Admission-part time or full time	Enclosed
5	Copies of RDC Approval letters of Ph.D	NA
6	Name of guide/supervisors with mobile number, email id	Enclosed
7	Copies of guide allotment letter	Enclosed
8	No. of increments sanctioned for Ph.D.	03
9	Published Research article-copies.	Enclosed.
10	Original Ph.D Thesis.- Book.	Available in office



Signature

Name & Designation

Dr. B. Prabhakar  
Asst. Prof of Mathematics

OU 772144

# Osmania University



Faculty of Science

This is to certify that Bellapuu Prabhakar  
son / daughter of Rajaram  
having pursued a course of study prescribed by this University  
and having passed the requirements by Examination and by  
thesis has been admitted to the Degree of

**Doctor Of Philosophy**

in the Subject of Mathematics

The title of the Thesis is :

Heat and Mass Transfer Analysis of MHD non-Newtonian Nanofluid over  
a Stretching surface

The candidate has been declared qualified for the award of the  
Degree of Ph.D. on 26 Aug 2017

Given under the seal of the University



CN061981639

Hyderabad August 27, 1941

Dated

June 17, 2019



  
Vice-Chancellor





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OSMANIA UNIVERSITY  
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Dated: 26 Aug, 2017

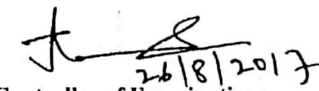
PRESS NOTE

The following candidates who had presented the Thesis on the subject mentioned against each for the degree of Ph.D are declared qualified for the award of Degree of Doctor of Philosophy (Ph.D.) of Osmania University, Hyderabad.

Ph.D.

S.N	Reference No.	Name of the Candidate/ Father Name	Subject	Thesis Title	Supervisor/ Regn. Date
1	PHD41241	Ms. Heera Bai D/o. Ramesh Kumar	Hindi(Oriental)	Viveki Roj Ke Katha Sahitya Ka Samaj Sastriya Adhyayan	Dr. T. Meena Kumari Principal, Pandit Narendra Oriental College, Narayanguda, Hyd (02/03/2009)
2	PHD41242	Mr. S Yadagiri S/o. Rajaiah	Telugu	Telangana Paatollo Samajika Chitrana	Dr. D. Balaiah Govt. Degree College, Siddipet (19/01/2009)
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6	PHD41246	Mr. Kandikonda Yadagiri S/o. Sambaiah	Telugu	Distinguished the Role of Situational Songs in Telugu Cinema	Prof. T. Kishen Rao (Retd.) Dept. of Telugu, OU (20/06/2013)
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8	PHD41248	Mr. Kundhojwala Krishna Kumar S/o. Mani Shankar	Geology	Hydrogeological Studies in Parts of Dindi River Basin, Mahabubnagar District, Telangana State, India	Dr. B. Linda Prabhakar Babu Dept. of Geology, OU (24/01/2009)

S.N	Reference No.	Name of the Candidate/ Father Name	Subject	Thesis Title	Supervisor/ Regn. Date
9	PHD41249	Mr. Bettapuu Prabhakar S/o. Rajaram	Mathematics	Heat and Mass Transfer Analysis of MHD non-Newtonian Nanofluid over a Stretching surface	Prof. Shankar Bandari Dept. of Mathematics, OU (26/10/2010)
10	PHD41250	Mr. Mohd Mahmood Ali S/o. Mohd Shabbir Ali	Computer Science & Engineering	Surveillance of Microblogs in Instant Messaging Applications	Prof. Lakshmi Rajamani (Retd.) Dept. of CSE, OU (22/01/2009)
11	PHD41251	Ms. Savaram Padmaja D/o. S Veera Sankara Rao	Computer Science & Engineering	Extracting Voter Sentiment for Indian General Election Forecast Using NLP Techniques	Prof. S. Sameena Fatima Dept. of CSE, UCE, OU (18/03/2011)
12	PHD41252	Mr. Erukala Yadaiahgoud S/o. E.Chandraiah	Chemistry	Synthesis and Biological Evaluation of Novel Heterocyclic Ring Pendent/Annulated Coumarins	Prof. Y. Jayaprakash Rao Dept. of Chemistry, OU (18/03/2011)
13	PHD41253	Mr. J Laxmikanth Rao S/o. J Narsingh Rao	Hindi	Sri Narayandas Jaju Ki Gazaloin Mein Yugh Bodh	Prof. Durgesh Nandini Head, Dept. of Hindi, OU (25/02/2013)
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16	PHD41256	Mr. Devender D S/o. Rajaiah	Economics	Marketing of Paddy in Telangana : A Study in Karimnagar District	Prof. M. Ramulu Dept. of Economics, OU (21/04/2008)

  
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26-8-17

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15. The Secretary, UGC, 35, Feroz Shah Road, New Delhi-110002
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17. The Senior Statistical Officer, U G C, (Info & Stats Bureau), 35, Feroz Shah Road, New Delhi - 110 221.
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19. The Examiner

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Dr. M. Rangamma  
Professor & Head



Department of Mathematics  
University College of Science,  
Osmania University,  
Hyderabad-500007.  
Phone: 27095083, 27682389.  
Email: headmath1ou@yahoo.co.in,  
headmathou@rediffmail.com.

Date: 31-12-2015

CERTIFICATE

This is to certify that Mr. B.Prabhakar Research Scholar working under the Supervision of Prof. B. Shankar has given the 1st Pre-Viva Seminar talk on his Ph.D. work entitled "Study of heat and mass transfer on MHD flows over stretching sheet" on 31-12-2015 at 2.30 PM in the Department of Mathematics.

Chairperson  
BoS in Mathematics

~~Chairman,~~  
Board of Studies in Mathematics  
Osmania University,  
Hyderabad-500007

Head 31-12-2015

Head  
Department of Mathematics  
Osmania University  
Hyderabad - 500 007

Dr. M. Rangamma  
Professor & Head




Department of Mathematics  
University College of Science,  
Osmania University,  
Hyderabad-500007.  
Phone: 27095083, 27682389.  
Email: headmathlou@yahoo.co.in,  
headmathou@rediffmail.com.

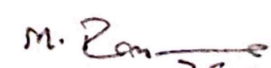
Date: 28-4-2016

CERTIFICATE

This is to certify that **Mr. B. Prabhakar**, Research Scholar working under the Supervision of **Prof. B. Shankar**, has given the Second Pre-Viva Seminar talk on his/her Ph.D. work entitled **Study of Heat and Mass Transfer on MHD flows over a stretching Sheet** on 28-4-2016 in the Department of Mathematics.

  
Chairperson  
BOS in Mathematics

Chairman  
Board of Studies in Mathematics,  
Osmania University,  
Hyderabad-500007.

  
Head <sup>28-4-2016</sup>  
Head  
Department of Mathematics  
Osmania University,  
Hyderabad - 500 007



JOINING REPORT OF PH.D./M.PHIL. COURSE,  
FACULTY OF SCIENCE, OSMANIA UNIVERSITY

4

1. Name :: BETTAPU PRABHAKAR  
 2. Father's Name :: RAJARAM  
 3. Details of Scholarship if any :: CSIR-JRF  
 4. College/Institute at which the Candidate proposes to work :: University college of science  
 5. Full-Time / Part-Time :: Full-Time  
 6. Name of the Supervisor :: Prof. B. SHANKER  
 7. Date of joining :: 26.10.2010  
 8. Department :: Mathematics  
 9. Topic of Research :: Heat and Mass Transfer Effects on MHD Convective flows

To  
The Dean,  
Faculty of Science

// Through Proper Channel //

Ref. No. 1033 / DFSc / 10 dt. 29-9-2010

Sir,

I am herewith submitting my joining report today i.e. on 26.10.2010

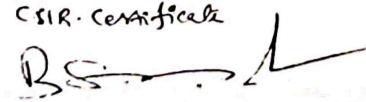
I have read the rules and regulations of the Ph. D. Course / M.Phil Course and I undertake to abide by them.


I understand that my admission may be cancelled, if the statements I made in my application are found to be false.

I have satisfied all conditions stipulated in my admission order and I am enclosing herewith the necessary certificates (if applicable).


LIST OF ENCLOSURES

1. D.D. No: 018107 Date 26.10.2010 Amount 2000/-  
 2. T.C, P.G. Memo  
 3. CSIR. Certificate

  
SIGNATURE OF SUPERVISOR

  
SIGNATURE OF CANDIDATE

SIGNATURE OF THE HEAD OF  
DEPARTMENT IN WHICH CANDIDATE  
PROPOSES TO WORK

  
SIGNATURE OF THE HEAD  
OF THE UNIV. DEPARTMENT

  
SIGNATURE OF THE DEAN

**DEAN**  
Faculty of Science  
Osmania University.  
HYDRABAD-500 007.

26/10/10  
26/10/10  
26/10/10



OFFICE OF THE DEAN  
FACULTY OF SCIENCE  
UNIV. COLLEGE OF SCIENCE  
OSMANIA UNIVERSITY  
HYDERABAD - 500 007

No. 920/DFSc/ 2012


Date: 11 2 JUN 2012

**ORDERS**

Sub: Change of status from Full-Time to Part -Time - Reg.

...

✓ On the recommendation of Head and Departmental Research Committee, Department of Mathematics, O.U. sanction is hereby accorded for Mr. B.Prabhakar, Ph.D. Research Scholar in Mathematics working under the supervision of Dr. B.Shanker to change his status from Full time to Part-time Research scholar as per the Ph.D. Rules and Regulations.

  
Dean  
Faculty of Science, O.U.N  
OSMANIA UNIVERSITY,  
HYDERABAD-500 007.

To

The person concerned.

Copy to:

1. The Head, Dept. of Mathematics, O.U.
2. The Addl. Controller of Examinations (Confidential), O.U.





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UNIV. COLLEGE OF SCIENCE  
OSMANIA UNIVERSITY  
HYDERABAD - 500 007

1033

No. /DFSc/OU/2010

Date: 29.9.2010

**ORDERS**

**Sub:** Ph.D. Admissions under UGC/CSIR/NET/M.Phil. candidates with Fellowships -2009 - 2010 Orders - Issued - Reg.

**Ref:** 1. Admission Notification No:734/DFSc/OU/2010 dated 6.7.2010 issued by the Dean, Faculty of Science, OU for UGC/CSIR/NET/M.Phil. candidates with Fellowships.  
2. Lr.No:963/F/2010/Acad.III dt.24.9.2010 from Deputy Registrar (Acad),OU

...

The candidates in the enclosed list are provisionally admitted to the Ph.D. course of Osmania University for the academic year 2009-2010 on the recommendation of the Admission Committee in the Faculty of Science in the subject mentioned against his/her name.

The selected candidates are required to fulfill the conditions, if mentioned against their names, and to submit their Joining Reports (Proforma provided), by **30.10.2010** failing which their admission orders would be deemed to have been withdrawn. No further notice will be given. The Joining Reports along with the original D.D. and all necessary documents duly forwarded by the Supervisor(s) and the concerned Head of the Department of Osmania University should be submitted by the prescribed date to the Dean, Faculty of Science, Osmania University. No joining report will be accepted without the T.C. (Transfer Certificate) in original or a letter from the respective University where from the Post Graduate Degree has been obtained to the effect that no separate Transfer Certificate will be issued by that University. A duly attested copy of the Migration Certificate wherever applicable should be submitted along with the Joining Report. The Dean's office shall then issue a list of names of the admitted candidates to the Heads of the Departments concerned, which shall be final.

The registration is valid for a period of four years for Full Time Research Scholars and five years for Part Time Research Scholars from the date of joining after which period it will be cancelled unless otherwise extended.

All the selected candidates both Full-Time and Part-Time have to pay the fee as under:

- |  |                  |
|--|------------------|
| 1. Both Full Time and Part Time Scholars working in the Osmania University | Rs.2000 per year |
| 2. Scholars working in recognised Research Centres outside the University  | Rs.5000 per year |

(P.T.O.)

through a demand draft in favour of "Dean, Faculty of Science, Osmania University". They should submit their Joining Reports in the concerned University Department in the prescribed proforma in triplicate along with the Original DD, and M.Sc., Certificate (Xerox Copy) in proof of satisfaction of the conditions stipulated.

If the candidate fails to pay the fees mentioned above within the specified time his/her admission will be cancelled without further notice to the candidate.

The selected candidates are required to submit an undertaking to the effect that they do not ask for hostel facilities (Annexure II) along with their joining reports, failing which they will not be granted admission.

Candidates selected under the category "Part Time" are required to submit an undertaking in triplicate on the proforma provided (Annexure-III) that they would be taking necessary leave as per rules of the University.

Their admission is conditional upon realization of dues to the University if any from the candidates. The admissions are made on the basis of the present occupation of the candidates. In case there is a change in occupation or place of work during the period of their candidature in the Ph.D., course, their admission is liable to be cancelled. Any change in their occupation should be brought to the notice of the Dean, through the Supervisor and the Head of the Department, and the Dean may permit the candidate to continue his/her Ph.D. course as per the rules.

The candidates who are admitted to the Ph.D. course shall not pursue any other course or appear for any other examination leading to any other Degree (both Full-Time and Part-Time) of this University or any other University. Any violation of this regulation will lead to the cancellation of admission.

S. N.

DEAN 28.9.10

Faculty of Science, O.U.

To

The Research Scholar concerned.

Copy forwarded for information and necessary action to:-

1. Principal, University College of Science, O.U.
2. The Vice Principal, Hostels, Univ. College of Science, O.U.
3. The Head, Department of \_\_\_\_\_, O.U.
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7. The Asst. Registrar (Academic), O.U.
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11. The P.A. to Registrar, O.U.
12. The Chief Warden, Hostels & Messes, O.U.
13. All Constituent Colleges / Research Centres concerned

DEAN  
Faculty of Science  
OSMANIA UNIVERSITY,  
HYDERABAD-509 007,



OFFICE OF DEAN, FACULTY OF SCIENCE, OU

LIST OF CANDIDATES SELECTED FOR ADMISSION INTO P.H.D. FOR THE YEAR 2009-2010

S.N	Name of the Candidate	Department	Name of the Supervisor	Status	Place of Work
1	Srinivas Majeti	Mathematics	Dr. B. Surender Reddy	Full Time	Osmania
2	Imtiyaz Md.	Mathematics	Dr. B. Krishna Reddy	Full Time	Osmania
3	N. Vijaya Bhaskar Reddy	Mathematics	Dr. N. Kishan	Full Time	Osmania
4	Bettapuu Prabhakar	Mathematics	Dr. B. Shankar	Full Time	Osmania
5	Srinivas Resddy C.	Mathematics	Dr. N. Kishan	Full Time	Osmania
6	V. Meenakshi	Mathematics	Dr. N. Kishan	Full Time	Osmania

S. R.

DEAN

28-9-10

Faculty of Science  
OSMANIA UNIVERSITY,  
HYDERABAD-500 007.

# MHD Stagnation Point Flow of a Casson Nanofluid Towards a Radially Stretching Disk with Convective Boundary Condition in the Presence of Heat Source/Sink

Besthapu Prabhakar<sup>1,\*</sup>, Shankar Bandari<sup>2</sup>, and Kishore Kumar<sup>2</sup>

<sup>1</sup>Department of Mathematics, Government Degree College, Peddapalli, Karimnagar 505172, Telangana, India

<sup>2</sup>Department of Mathematics, Osmania University, Hyderabad 500007, Telangana, India

This article deals the study of effects of convective condition on MHD stagnation point Casson nanofluid flow due to radially stretching disk. The system of partial differential equations governs the flow that transformed into the nonlinear ordinary differential equations by employing suitable similarity transformations. The resulting system of ODE's is successfully solved numerically by Runge-Kutta fourth order method along with shooting technique. The effects of various emerging parameters on velocity, temperature and concentration profile are discussed in detail and presented through graphically. Velocity profile decreases by increasing Hartman number and Casson fluid parameter. Nanoparticle concentration increases with increasing thermophoresis parameter, but reverse trend is observed with the effect of Brownian motion parameter.

**KEYWORDS:** Stagnation Point Flow, Convective Condition, Radially Stretched Disk, Casson Nanofluid, Viscous Dissipation.

## 1. INTRODUCTION

Boundary layer flow induced by a continuous stretching sheet gained considerable attraction in the past few decades due its extensive applications in many engineering processes. Some examples of practical applications of moving stretching surfaces are extraction of polymer sheet, wire drawing, paper production, glass-fiber production, hot rolling, solidification of liquid crystals, petroleum production, continuous cooling and fibers spinning, exotic lubricants and suspension solutions. In particular the radial stretching occurs during the expansion of balloons. In view of these applications Sakiadis<sup>1,2</sup> initiated the study of boundary layer flow over continuous stretching sheet moving with a constant speed. Crane<sup>3</sup> obtained an elegant analytical solution for two dimensional flow induced by a stretching sheet which moves with a velocity varying linearly with the distance from a fixed point. Due to various numerous and industrial applications Crane's problem has been considered by researchers under different physical effects and different sheets. Various researchers have explored the flow over stretching surfaces in different

circumstances. Munawar et al.<sup>4</sup> presented an analytic solution of flow of viscous fluid between two stretching disks with slip boundaries. Hayat et al.<sup>5</sup> investigated flow and heat transfer in second grade fluid over a stretching sheet subjected to convective boundary conditions. Ashraf and Batool<sup>6</sup> carried out a numerical study of an axisymmetric steady laminar incompressible flow from an electrically conducted micropolar fluid over a stretching disk.

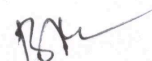
In fluid mechanics the stagnation point is the location where the local velocity tends to zero i.e., field of a flow regarding the body and the fluid particles has zero velocity with respect to the body. Stagnation points exist at the surface of objects in the flow field, where the fluid is brought to the rest by the object. The highest fluid pressure, heat transfer and rate of mass deposition are occurred in the stagnation region. Stagnation point flow analysis is very important both in natural and industrial phenomena. Some of the examples of stagnation point flow are flows over the tips of submarines, oil ships, rockets and aircrafts. An interesting example of stagnation point flow is the blood flow at a junction within an artery. Hiemenz<sup>7</sup> was first to study two-dimensional stagnation flow using a similarity transform to reduce the Navier–Stokes equations to nonlinear ordinary differential equations. The axisymmetric case was solved by Homann.<sup>8</sup>

\*Author to whom correspondence should be addressed.

Email: [prabhakarbesthapu@gmail.com](mailto:prabhakarbesthapu@gmail.com)

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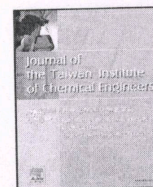






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Short communication

## Mixed convection flow of thermally stratified MHD nanofluid over an exponentially stretching surface with viscous dissipation effect

Prabhakar Besthapu<sup>a</sup>, Rizwan Ul Haq<sup>b,\*</sup>, Shankar Bandari<sup>a</sup>, Qasem M. Al-Mdallal<sup>c</sup><sup>a</sup> Department of Mathematics, Osmania University, Hyderabad, India<sup>b</sup> Department of Electrical Engineering, Bahria University, Islamabad Campus, Islamabad, Pakistan<sup>c</sup> Department of Mathematical Sciences, UAE University, P.O. Box 15551, Al Ain, United Arab Emirates

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## ABSTRACT

The present analysis concentrates to examine the influence of both thermal and solutal stratification on magneto-hydrodynamics (MHD) nanofluid flow along an exponentially stretching sheet. Moreover, simultaneous effects of mixed convection and viscous dissipation are also analyzed to determine the thermal conductivity within the restricted domain. Energy and concentration equation consist of two important slip mechanisms, namely: the Brownian motion of nanoparticles and the thermophoresis due to concentration difference. By the mean of compatible similarity transformed, a system of PDEs is converted into the system of nonlinear ODEs. The resulting nonlinear ODEs are successfully solved via the implicit finite difference method (FDM). Obtained numerical solutions are plotted for each profile for different and converging values of including parameters. To validate the results, numerical values of Nusselt number are compared with the existing literature for a particular case. Obtained results present the significant impact of each parameter on temperature and concentration. Nanofluid flow behaviour is also observed via velocity profile.

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## 1. Introduction

Fluid flow past along a rough or flat surfaces attain a considerable attention by virtue of its extensive and considerable applications in engineering and manufacturing processes. Few important and foremost useful examples related to applications include the polymer extrusion, wires and fibre undercoat, food stuff wrapping, manufacturing of bags and papers, and petroleum manufacturing goods. Initially, Sakiadis [1] described the idea of boundary layer theory along a uniform moving surface that deals the free stream velocity and ambient fluid temperature is considered to be zero. Later on, Crane [2] has been examining the work for linear stretching velocity and it is proportional to the distance from the fixed slit. Thereafter the revolutionary idea presented by Crane has been extended by several other researchers by considering the various effects of heat and mass flow [3–5]. In the current decay, the most significant part of literature of fluid models is dealing with the linear stretching surface with various physical phenomena. However, Gupta and Gupta [3] established that all the physical phenomena need not to be dealt with linear but it can be dealt with the

exponential or non-linear stretching for both heat and mass transfer. They have further discovered that sheet can be considered as a permeable to deal the phenomena of suction and injection of the fluid.

Apart from all the fundamental problem of linear stretching sheet, existing literature also witnesses that flow examination due to exponentially stretching sheet is also an important factor in most of the manufacturing process. In the beginning, the idea of flow past over an exponentially stretching sheet is presented by Magyari and Keller [6]. They have found the numerical solution of fluid flow over a sheet that is stretched with exponential velocity that deals the phenomena of heat and mass transfer characteristics. In another study, Elbashareshy [7] extend the idea of Magyari and Keller [6] for exponential stretching sheet to deal the heat transfer with suction/injection effect. Currently many researchers have considered different and important ideas to deal the flow over an exponentially stretching surface for both Newtonian and non-Newtonian fluid models [8–15].

In the recent decay, heat transfer is one of the essential key features in the energy development at the industrial level and manufacturing process of any equipment. Despite of that fact heat addition, removing or transfer from one place to another place during the manufacturing process is totally based upon the thermal performance of working fluid. In several cases: water, engine oil,

\* Corresponding author.

E-mail addresses: [r.haq.qau@gmail.com](mailto:r.haq.qau@gmail.com), [ideal\\_riz@hotmail.com](mailto:ideal_riz@hotmail.com) (R.U. Haq), [q.almdallal@uaeu.ac.ae](mailto:q.almdallal@uaeu.ac.ae) (Q.M. Al-Mdallal).<http://dx.doi.org/10.1016/j.jtice.2016.12.034>

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# Effects of Inclined Magnetic Field and Chemical Reaction on Flow of a Casson Nanofluid with Second Order Velocity Slip and Thermal Slip Over an Exponentially Stretching Sheet

Besthapu Prabhakar<sup>1,2</sup> · Shanker Bandari<sup>2</sup> ·  
Ch. Kishore Kumar<sup>2</sup>

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**Abstract** The numerical investigation of the flow of a Casson nanofluid over an exponential stretching sheet with the effects of the inclined magnetic field and chemical reaction is presented. Moreover we considered second order velocity slip and thermal slip. The basic governing partial differential equations are converted into nonlinear ordinary differential equations by employing suitable similarity transformations. The resulting equations are successfully solved by using an implicit finite difference scheme known as Keller-Box method. Comparisons of numerical results have been done with previously published results and found in good agreement. The effects of various non dimensional parameters (magnetic parameter, aligned angle, Casson parameter, second order velocity slip, thermal slip, Prandtl number, Thermophoresis parameter, Brownian motion parameter, Lewis number, chemical reaction parameter) on velocity, temperature and concentration are discussed in detail and presented through graphs. It is observed that the increase in aligned angle decreases the velocity profile. Considerable slip effects are found on velocity and temperature.

**Keywords** Exponential stretching sheet · Inclined angle · Casson fluid · Second order velocity slip · Thermal slip · Chemical reaction

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✉ Besthapu Prabhakar  
prabhakarbesthapu@gmail.com

Shanker Bandari  
bandarishanker@yahoo.co.in

Ch. Kishore Kumar  
kishoresai09@gmail.com

<sup>1</sup> Department of Mathematics, Govt Degree College, Peddapalli, Karimnagar, Telangana, India

<sup>2</sup> Department of Mathematics, Osmania University, Hyderabad, Telangana, India

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# Impact of inclined Lorentz forces on tangent hyperbolic nanofluid flow with zero normal flux of nanoparticles at the stretching sheet

Besthapu Prabhakar<sup>1,2</sup> · Shankar Bandari<sup>2</sup> · Rizwan Ul Haq<sup>3</sup>

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**Abstract** This framework is devoted to analyze the tangent hyperbolic fluid in the presence of nanoparticles. In order to disperse the nanoparticle from the surface of sheet, condition of zero normal flux of nanoparticles is introduced at the surface. Inclined magnetic field is applied with an aligned angle  $\gamma$  at the surface of the sheet. Moreover, consideration of nanoparticles which are passively controlled at the surface is physically more realistic condition. The system of partial differential equations generated for tangent hyperbolic nanofluid are modeled and then converted into the system of nonlinear ordinary differential equations by employing suitable similarity transformations. Obtained systems of ordinary differential equations along with the condition of zero normal flux are successfully solved numerically by Runge–Kutta fourth-order method with shooting technique. The effects of various emerging parameters on velocity, temperature and concentration profiles are discussed in detail and presented graphically. Variation of skin friction coefficient and local Nusselt number are also oriented to analyze the variation of nanofluid at the surface. Considerable effects are found on velocity, temperature and concentration with the variable values of Weissenberg number  $We$  and inclination of angle  $\gamma$ . It is finally concluded that increase in the Weissenberg

number and power law index reduce the velocity profile; however, thermophoresis parameter shows the dominant effects on temperature and concentration profile. Significant effects on velocity, temperature and concentration profiles are also determined for both suction and injection cases.

**Keywords** Non-Newtonian · Nanofluid · Aligned angle · Tangent hyperbolic fluid · Thermophoresis · Brownian motion

## 1 Introduction

The dynamics of non-Newtonian fluids have extensive applications in chemical engineering and industrial manufacturing process. Particularly, flow and heat transfer of rheological fluid perform a significant part in food engineering, petroleum production, power engineering and solutions, and polymer melt in the plastic processing industries. There are non-Newtonian fluids in nature with various properties. The classical Navier–Stokes equations are inadequate to understand the typical properties of non-Newtonian fluids. In view of this, various constitutive equations of non-Newtonian fluids have been proposed. There is no unique non-Newtonian fluid model that predicts entire characteristics of non-Newtonian fluids. Among various models, one of the important branches of non-Newtonian fluid is hyperbolic tangent fluid which is useful for chemical engineering systems and it has the capacity to describe shear thinning phenomena. This model has several advantages compare to other non-Newtonian models, includes simplicity, computational easiness and physical robustness. Examples of tangent

✉ Rizwan Ul Haq  
ideal\_riz@hotmail.com; r.haq.qau@gmail.com

<sup>1</sup> Department of Mathematics, Govt. Degree College, Peddapalli, Karimnagar, Telangana, India

<sup>2</sup> Department of Mathematics, Osmania University, Hyderabad, Telangana, India

<sup>3</sup> Department of Electrical Engineering, Bahria University, Islamabad Campus, Islamabad 44000, Pakistan



# Thermal radiation and slip effects on MHD stagnation point flow of non-Newtonian nanofluid over a convective stretching surface

Prabhakar Besthapu<sup>1</sup> · Rizwan Ul Haq<sup>2</sup> · Shankar Bandari<sup>1</sup> · Qasem M. Al-Mdallal<sup>3</sup>

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**Abstract** The present analysis examines the combine effects of thermal radiation and velocity slip along a convectively nonlinear stretching surface. Moreover, MHD effects are also considered near the stagnation point flow of Casson nanofluid. Slipped effects are considered with the porous medium to reduce the drag reduction at the surface of the sheet. Main structure of the system is based upon the system of partial differential equations attained in the form of momentum, energy, and concentration equations. To determine the similar solution system of PDEs is rehabilitated into the set of nonlinear ordinary differential equations (ODEs) by employing compatible similarity transformation. Important physical parameters are acquired through obtained differential equations. To determine the influence of emerging parameters, resulting set of ODE's in term of unknown function of velocity, temperature, and concentration are successfully solved via Keller's box-scheme. All the obtained unknown functions are discussed in detail after plotting the results against each physical parameter. To analyze the behavior at the surface: skin friction, local Nusselt and Sherwood numbers are also illustrated against the velocity ratio parameter  $A$ , Brownian motion  $Nb$ , thermophoresis  $Nt$ , and thermal radiation parameters  $R$ . Results obtained

from the set of equations described that skin friction is decreasing function of  $A$ , and local Nusselt and Sherwood number demonstrate the significant influenced by Brownian motion  $Nb$ , thermophoresis  $Nt$ , and radiation parameters  $R$ .

**Keywords** Axisymmetric · Radially stretched · Stagnation point · Casson nanofluid · Convective condition · Slip condition

## 1 Introduction

Boundary layer flow induced by a continuous stretching sheet gained considerable attraction in the past few decades due its extensive applications in many engineering processes. Some examples of practical applications of moving stretching surfaces are wire illustration, paper and sheet production, hot rolling materials, solidification of liquid crystals, daily usage goods in kitchen, glass and fiber production, etc. In the light of above said application, initially Sakiadis [1, 2] demonstrates the application of boundary layer flow for continuous stretching sheet that is moving with a uniform speed. After that, Crane [3] reported an elegant analytical solution for boundary layer phenomena induced due to a stretching sheet. Due to various numerous and industrial applications, Crane's work has been considered by various researchers under various physical aspects and different sheets. Currently, few more usable flows that past over a stretching sheet with difference ratio are exponential, nonlinear, quadratic, and oscillatory [4–7] are under consideration in current era.


In fluid mechanics, the stagnation point is the location where the local velocity tends to zero. Usually, stagnation points appear at the surface of any object in the flow field, where velocity of the fluid becomes zero due to that

✉ Rizwan Ul Haq  
rizwanulhaq.buic@bahria.edu.pk

<sup>1</sup> Department of Mathematics, Osmania University,  
Hyderabad, Telangana, India

<sup>2</sup> Department of Electrical Engineering, Bahria University,  
Islamabad 44000, Pakistan

<sup>3</sup> Department of Mathematical Sciences, UAE University, Al  
Ain P.O. Box 15551, United Arab Emirates





# A REVISED MODEL TO ANALYZE MHD FLOW OF MAXWELL NANOFLUID PAST A STRETCHING SHEET WITH NONLINEAR THERMAL RADIATION EFFECT

Besthapu Prabhakar,<sup>1,\*</sup> Shanker Bandari,<sup>2</sup> & Cherlacola Srinivas Reddy<sup>3</sup>

<sup>1</sup>Kakatiya Government College, Hanamkonda, Telangana, 506001, India

<sup>2</sup>Department of Mathematics, Osmania University, Hyderabad, Telangana, 500007, India

<sup>3</sup>Government Degree College, Mulugu, Telangana, 506343, India

\*Address all correspondence to: Besthapu Prabhakar, Kakatiya Government College, Hanamkonda, Telangana, 506001, India, E-mail: prabhakarbesthapu@gmail.com

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*This article reports the magnetohydrodynamic flow of a Maxwell nanofluid over a stretching sheet under the influence of nonlinear thermal radiation. A revised model in which mass flux of nanoparticles is zero on the surface is implemented to attain physically applicable results. For passively controlled mass flux, the nanoparticle volume fraction is defined separately by the temperature gradient, resulting in zero nanoparticle flux at the surface. Additionally, the influence of nonlinear Rosseland radiation is delved. The modeled partial differential equations are transformed to nonlinear ordinary differential equations by utilizing appropriate similarity transformations. The resulting equations are solved numerically using the spectral quasi-linearization method. To visualize the impact of various controlling parameters on velocity, temperature, and concentration profiles, graphs have been plotted. It is observed that growing values of Maxwell parameter lead to attenuation in the velocity profile, but the reverse trend is observed in temperature and concentration profiles.*

**KEY WORDS:** stretching sheet, nonlinear radiation, Maxwell parameter, zero normal flux, Brownian motion

## 1. INTRODUCTION

In recent years, investigations into non-Newtonian fluid flows have been significantly enhanced because of the fluids' extensive practical implications in industrial and manufacturing processes. Some materials, including muds, slurries, blood, shampoos, fruit juices, and printing inks, exhibit characteristics of non-Newtonian fluids. Since a single model is inadequate to describe all characteristics of non-Newtonian fluids, researchers have proposed various models to study such fluids. The majority of simple non-Newtonian fluid models, including second-grade fluids (Hayat et al., 2004; Rajagopal, 1982; Bandelli, 1995; Tan and Masuoka, 2005; Sadeghy and Sharifi, 2004; Serdar and Salih Dokuz, 2006; Sajid et al., 2009), are insufficient to study the flows of highly elastic fluids (polymer melts), and the viscosity in these models is not shear dependent. Moreover, these non-Newtonian fluid models are unable to predict the effects of stress relaxation. These models are mainly classified into three categories: differential-, rate-, and integral-type models. Maxwell fluid models are a subclass of rate-type non-Newtonian models in which relaxation time effects are taken into consideration. This model can easily predict the characteristics of the relaxation time. Such effects cannot be described in the differential-type non-Newtonian fluids. This fluid model is especially useful for polymers of low molecular weight.

In the beginning, Harris (1977) derived the boundary layer equations for two-dimensional flow of upper-convected Maxwell fluid. Sadeghy et al. (2005) studied the Sakiadis flow of an upper-convected Maxwell fluid. In this study, they concluded that velocity decreases with respect to the local Deborah number. Magnetohydrodynamic (MHD) flow



## RESEARCH ARTICLE

# Heat transfer analysis of inclined magnetic field and activation energy in Maxwell nanofluid with thermophoresis effects

Besthapu Prabhakar<sup>1</sup>  | Fazle Mabood<sup>2</sup> 

<sup>1</sup>Department of Mathematics, Kakatiya Government College, Hanamkonda, Telangana, India

<sup>2</sup>Department of Information Technology, Fanshawe College, London, Ontario, Canada

**Correspondence**

Besthapu Prabhakar, Department of Mathematics, Kakatiya Government College, Hanamkonda, Telangana 506001, India.  
Email: prabhakarbesthapu@gmail.com

**Abstract**

Numerical analysis is performed for incompressible Maxwell nanofluid model flow under the implications of thermophoresis and inclined magnetic field over a convectively stretched surface. The system that comprises differential equations of partial derivatives is remodeled into the system of ordinary differential equations via similarity transformations and then solved through by Runge–Kutta–Fehlberg with shooting technique. The physical parameters, which emerge from the derived system, are discussed in graphical formats. Excellent proficiency in the numerical process is analyzed by comparing the results with available literature in limiting scenarios. The significant outcomes of the current investigation are that the velocity field decays for higher fluid parameters while that peter out the fluid temperature. Further, the heat transfer rate is reduced with the incremental values of fluid and thermophoresis parameters while it uplifts with Biot and Prandtl numbers.

**KEYWORDS**

activation energy, Maxwell fluid, MHD, nanofluid, thermophoresis



## 1 | INTRODUCTION

During the past decade, the study of non-Newtonian liquids has remarkably attracted the attention of researchers and scientists due to its practical implications in manufacturing and



## Impact of Activation Energy in Darcy-Forchheimer Flow of Cross Nanofluid over a Radial Stretching Surface with Viscous Dissipation and Joule Heating



Cherlacola Srinivas Reddy<sup>1</sup>, Besthapu Prabhakar<sup>2\*</sup>

<sup>1</sup> Department of Mathematics, ABV Government College, Janagaon 506167, Telangana, India

<sup>2</sup> Department of Mathematics, Kakatiya Government College, Hanamkonda 506001, Telangana, India

Corresponding Author Email: [prabhakarbesthapu@gmail.com](mailto:prabhakarbesthapu@gmail.com)

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### ABSTRACT

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#### Keywords:

*Darcy-Forchheimer flow, cross fluid, joule heating, viscous dissipation, radially stretching surface*

This framework analyzes the impact of activation energy (AE) and binary chemical reaction (BCR) in Darcy-Forchheimer flow of cross fluid with nanoparticles due to radially stretched surface. Moreover slip, joule heating and viscous dissipation aspects have been considered. Ordinary differential equations acquired from the modelled governing partial differential equations with the assistance of suitable transformations. Further the system of nonlinear equations is computed numerically by Runge-Kutta-Fehlberg method cum shooting technique. Graphical representation has been given to analyze the velocity, temperature and concentration fields with the effect of various pertinent parameters. It is evident that inertia coefficient declines the velocity. Velocity decays for larger Weissenberg number while opposite trend observed in temperature field. Temperature field rises for augmented values of Eckert number. Concentration increases with increase of energy parameter.

### 1. INTRODUCTION

The study of non-Newtonian liquids flow and their features grab the attention of many researchers and scientists due to their significant applications in industrial products and procedures. Non-Newtonian liquids possess the non-linear relation between stress and strain. Depends on high viscosity few commonly used non-Newtonian fluids are toothpaste, lubricants, ketchup, silly putty, syrup, honey, paint, plastic, etc. Investigation of non-Newtonian flow field and its characteristics is quite difficult as compared to Newtonian liquids. To describe the features of non-Newtonian fluid flow by the well known Navier-Stokes equations are inadequate. Therefore, various fluid models have been suggested to demonstrate the features of non-Newtonian fluids. Cross [1] proposed Cross fluid model which is a major subclass of generalized Newtonian fluids and it predicts the shear thinning effects for both high and low shear rates. This fluid model has remarkable applications in engineering calculations due to the existence of the time constant. The empirical study of the cross fluid model was presented by Escudier et al. [2] by considering Cross equation with the non-Newtonian liquids carboxy methyl cellulose (CMC), xanthan gum (XG) and illustrated the fluid flow data. Xie and Jin [3] analyzed the free surface flow of the non-Newtonian fluid by WC-MPS method to find the four rheology parameters of Cross fluid model. Khan et al. [4] considered Cross fluid flow and heat transfer over a linear stretching surface and employed a numerical technique to solve boundary layer equations of the problem. From this study it is noticed that velocity profile is decreased with enlarging the Weissenberg number whereas inverse trend identified for the temperature field. Fluid flows through porous media have numerous applications corresponding to chemical engineering and geophysical systems. Such applications may

include movement of water in reservoirs fermentation process, crude oil production, grain storage, ground water systems, ground water pollution and recovery systems etc. The Darcy's law is reasonable under circumstances of low velocity and little porosity but it is inadequate whenever inertial and boundary features occur at higher velocity. Under such conditions it is impossible to neglect the inertia and boundary impacts. The non-Darcian porous medium is the revised form of Darcy law which includes the inertia and boundary effects. For higher velocity flow, the Forchheimer [5] formula is used in which squared velocity term has been added in the equation to predict the behavior of inertia and boundary effects. Muskat [6] termed this as "Forchheimer factor" which always reasonable for large Reynolds number. Darcy-Forchheimer flow of non-newtonian fluid was analyzed by Seddeek [7]. Pal and Mondal [8] studied influence of the Lorentz forces in a non-Darcy flow by considering variable viscosity. Shehzad et al. [9] considered Cattaneo-Christov heat flux model and homogeneous-heterogeneous reactions in Darcy-Forchheimer flow of Oldroyd-B fluid. Their analysis reveals that temperature is diminishing for Cattaneo-Christov heat flux model when compared to classical Fourier's law of heat conduction. Hayat et al. [10] examined Cattaneo-Christov model in flow of Maxwell fluid through a non-Darcy porous medium with temperature-dependent thermal conductivity. Fluids cooling and heating is the major issue in many industrial fields such as power manufacturing and transportation. Efficient cooling techniques are required to cool any kind of high energy equipment. The working fluids which are utilized in the industries have poor thermal conductivity due to this they cannot meet modern cooling challenges. Therefore, to improve the heat transfer capabilities of common heat transfer fluids Choi and Eastman [11] proposed the idea of nanofluids which is mixture of tiny

# Heat and Mass Transfer Analysis of MHD non-Newtonian Nanofluid over a Stretching surface

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Under the Supervision of  
**Dr. Bandari Shankar**



**Department of Mathematics**  
**Osmania University, Hyderabad-07**  
**Telangana, India**  
**April 2017**



**Dr. B. Shankar**  
M.Sc., Ph.D., M.ISM, M.ISTE  
Dean Faculty of Science  
Professor of Mathematics



**University College of Science,  
Osmania University,  
Hyderabad- 500007, India**  
E-mail: bandarishanker@yahoo.co.in

## CERTIFICATE

This is to certify that the research work incorporated in this thesis entitled "**Heat and Mass Transfer Analysis of MHD non-Newtonian Nanofluid over a Stretching surface**" is done by Mr. Bettapuu Prabhakar for the award of Doctor of Philosophy in Mathematics at Osmania University, Hyderabad. It is a record of bonafide research work carried out by him under my supervision and guidance. The work in this thesis has not been submitted for the award of research degree to any other University.

Place: Hyderabad

Date: 21.8.17

(Prof. Bandari Shankar)